OFFICIAL SYLLABUS

464 - Partial Differential Equations

Adopted - Fall 2019 (Committee: Drs. Leem, Liu, Pelekanos)

Course Description: Partial differential equations, heat equation, wave equation, Laplace's equation, Fourier series, Fourier transform, Laplace transform, method of separation of variables.

Prerequisites: Math 223, 250, 305, and 321 with a grade of C or better, or consent of instructor.

Textbook: Applied Partial Differential Equations with Fourier Series and Boundary Value Problems, 5th edition, by Richard Haberman. ISBN 9780321797056.

Course objectives: At the conclusion of this course, students should be able to
1. Describe real-world models using PDEs.
2. Solve first order PDEs using the method of characteristics.
3. Determine the existence, uniqueness, and well-posedness of solution of PDEs.
4. Solve linear second order PDEs using canonical variables for initial-value problems, Separation of Variables and Fourier series for boundary value problems.

Course Outline: (Boxed sections are optional, each lecture covers about 1 section)

1. Heat Equation

1.2 Derivation of the Conduction of Heat in a One-Dimensional Rod

1.3 Boundary Conditions

2. Method of Separation of Variables

2.2 Linearity

2.3 Heat Equation with Zero Temperatures at Finite Ends

2.4 Worked Examples with the Heat Equation: Other Boundary Value Problems

2.5 Laplace's Equation: Solutions and Qualitative Properties

3. Fourier Series

3.2 Statement of Convergence Theorem

3.3 Fourier Cosine and Sine Series

3.4 Term-by-Term Differentiation of Fourier Series

3.5 Term-By-Term Integration of Fourier Series

3.6 Complex Form of Fourier Series

4. Wave Equation: Vibrating Strings and Membranes

4.2 Derivation of a Vertically Vibrating String

4.3 Boundary Conditions

4.4 Vibrating String with Fixed Ends

4.5 Vibrating Membrane

5. Sturm-Liouville Eigenvalue Problems

5.2 Examples

5.3 Sturm-Liouville Eigenvalue Problems

5.4 Worked Example: Heat Flow in a Nonuniform Rod without Sources

5.5 Self-Adjoint Operators and Sturm-Liouville Eigenvalue Problems

7. Higher Dimensional Partial Differential Equations

7.2 Separation of the Time Variable

7.3 Vibrating Rectangular Membrane

7.9 Laplace's Equation in a Circular Cylinder

8. Nonhomogeneous Problems

8.2 Heat Flow with Sources and Nonhomogeneous Boundary Conditions

8.3 Method of Eigenfunction Expansion with Homogeneous Boundary Conditions

8.4 Method of Eigenfunction Expansion Using Green's Formula

8.5 Forced Vibrating Membranes and Resonance

8.6 Poisson's Equation

10. Infinite Domain Problems: Fourier Transform Solutions

10.2 Heat Equation on an Infinite Domain

10.3 Fourier Transform Pair

10.4 Fourier Transform and the Heat Equation

10.5 Fourier Sine and Cosine Transforms: The Heat Equation on Semi-Infinite Intervals

10.6 Worked Examples Using Transforms

12. The Method of Characteristics for Linear Wave Equations

12.2 Characteristics for First-Order Wave Equations

12.3 Method of Characteristics for the One-Dimensional Wave Equation

12.4 Semi-Infinite Strings and Reflections

12.5 Method of Characteristics for a Vibrating String of Fixed Length

13. Laplace Transform Solution of Partial Differential Equations

13.2 Properties of the Laplace Transform

13.4 A Signal Problem for the Wave Equation

13.5 A Signal Problem for a Vibrating String of Finite Length