

**OFFICIAL SYLLABUS**  
**MATH 223-Logic and Mathematical Reasoning**

**Adopted Spring 2019**

(Committee: Drs. J. Loreaux, M.-S. Song and G. S. Staples)

**Catalog Description.** Concepts and techniques essential to advanced mathematics: logic, methods of proof, sets, relations, induction, functions, cardinality, combinatorics and graph theory.

**Prerequisites:** MATH 150 with grades of C or better.

**Textbook:** *Mathematical Proofs: A Transition to Advanced Mathematics*, 4<sup>th</sup> edition by Gary Chartrand Albert D. Polimeni and Ping Zhang ISBN: 978-0134746753

**Course Outline:**

**Chapter 1 Sets**

- 1.1 Describing a Set
- 1.2 Subsets
- 1.3 Set Operations
- 1.4 Indexed Collection of Sets
- 1.5 Partitions of Sets
- 1.6 Cartesian Products of Sets

**Chapter 2 Logic**

- 2.1 Statements
- 2.2 Negations
- 2.3 Disjunctions and Conjunctions
- 2.4 Implications
- 2.5 More on Implications
- 2.6 Biconditionals
- 2.7 Tautologies and Contradictions
- 2.8 Logical Equivalence
- 2.9 Some Fundamental Properties of Logical Equivalence
- 2.10 Quantified Statements
- 2.11 Characterizations

**Chapter 3 Direct Proof and Proof by Contrapositive**

- 3.1 Trivial and Vacuous Proofs
- 3.2 Direct Proofs
- 3.3 Proof by Contrapositive
- 3.4 Proof by Cases
- 3.5 Proof Evaluations

**Chapter 4 More on Direct Proof and Proof by Contrapositive**

- 4.1 Proofs Involving Divisibility of Integers
- 4.2 Proofs Involving Congruence of Integers

- 4.3 Proofs Involving Real Numbers
- 4.4 Proofs Involving Sets
- 4.6 Proofs Involving Cartesian Products of Sets

## **Chapter 5 Existence and Proof by Contradiction**

- 5.1 Counterexamples
- 5.2 Proof by Contradiction
- 5.3 A Review of Three Proof Techniques
- 5.4 Existence Proofs
- 5.5 Disproving Existence Statements

## **Chapter 6 Mathematical Induction**

- 6.1 The Principle of Mathematical Induction
- 6.2 A More General Principle of Mathematical Induction
- 6.3 The Strong Principle of Mathematical Induction
- 6.4 Proof by Minimum Counterexample

## **Chapter 7 Reviewing Proof Techniques**

- 7.1 Reviewing Direct Proof and Proof by Contrapositive
- 7.2 Reviewing Proof by Contradiction and Existence Proofs
- 7.3 Reviewing Induction Proofs
- 7.4 Reviewing Evaluations of Proposed Proofs

## **Chapter 9 Equivalence Relations**

- 9.1 Relations
- 9.2 Properties of Relations
- 9.3 Equivalence Relations
- 9.4 Properties of Equivalence Classes
- 9.5 Congruence Modulo  $n$
- 9.6 The Integers Modulo  $n$

## **Chapter 10 Functions**

- 10.1 The Definition of Function
- 10.2 One-to-one and Onto Functions
- 10.3 Bijective Functions
- 10.4 Composition of Functions
- 10.5 Inverse Functions

## **Chapter 11 Cardinalities of Sets**

- 11.1 Numerically Equivalent Sets
- 11.2 Denumerable Sets
- 11.3 Uncountable Sets
- 11.4 Comparing Cardinalities of Sets

## **Chapter 12 Proofs in Number Theory**

- 12.1 Divisibility Properties of Integers

- 12.2 The Division Algorithm
- 12.3 Greatest Common Divisors
- 12.4 The Euclidean Algorithm
- 12.5 Relatively Prime Integers

### **Chapter 13 Proofs in Combinatorics**

- 13.1 The Multiplication and Addition Principles
- 13.2 The Principle of Inclusion-Exclusion
- 13.3 The Pigeonhole Principle
- 13.4 Permutations and Combinations
- 13.6 The Binomial Theorem
- 13.7 Permutations and Combinations with Repetition

Course objectives:

At the conclusion of this course, students should be able to:

1. perform computations involving logic, sets, functions, and graphs.
2. construct proofs using the following techniques: direct, contrapositive, contradiction, induction, and combinatorial.
3. construct proofs involving basic number theory (divisibility, gcd, parity, rational numbers), sets, relations and partitions, functions (including injective and surjective properties), cardinality, and basic graph theory.
4. use counting techniques to count sets of moderate complexity.

**Any instructor should cover all of the material specified, additional sections are optional**